16. Capacitors
Problem set (suggested solutions)

1 **(a)** apply potential difference between the plates — M1

causes charge separation between the plates — A1

(b) **(i)** straight line starting at the origin — B1

line with positive gradient ending at (V, Q) — B1

(ii) work done is the area under the graph — B1

$W = \frac{1}{2} QV$ — B1

(c) **(i)** Electric field is constant / remains the same — B1

$$E = \frac{V}{L} = \frac{V_N}{L-D}$$ M1

$$V_N = \left(\frac{L-D}{L}\right) V$$

(ii) Charge on each plate remains the same. — B1

Charge on each plate $Q = CV$ — C1

$$C_N = \frac{Q}{V_N} = \frac{CV}{\left(\frac{L-D}{L}\right)V} = \left(\frac{L}{L-D}\right) C$$ A1

2 **(a)** Combined capacitance
$= 24 + (1/24 + 1/24 + 1/24)^{-1}$
$= 24 + 8$
$= 32\ \mu F$

(b) **(i)** parallel combination of two in series and a single capacitor

(ii) one capacitor in series with two in parallel

3 **(a)**

	relationship between charges	relationship between p.d.s
series	$Q_S = Q_1 = Q_2$	$V_S = V_1 + V_2$
parallel	$Q_S = Q_1 + Q_2$	$V_S = V_1 = V_2$

(b) **(i)** $E = ½ CV^2$

$19 \times 10^{-3} = ½ (470 \times 10^{-6})V^2$ C1

$V = 9.0$ V A1

(ii) $E = Q^2 / 2C$ (or use $C = Q / V$)

$19 \times 10^{-3} = Q^2 / (2 \times 470 \times 10^{-6})$ C1

$Q = 4.2 \times 10^{-3}$ C A1

(iii) **1.** total capacitance = $(470 + 180) \times 10^{-6} = 650 \times 10^{-6}$ C1

total charge unchanged C1

p.d. across capacitor
= p.d. across total capacitance
= $Q / C = (4.226 \times 10^{-3}) / (650 \times 10^{-6})$
= 6.5 V A1

2. Total energy in the two capacitors
= $Q^2 / 2C = (4.226 \times 10^{-3})^2 / (2 \times 650 \times 10^{-6})$
= 13.74 mJ C1

Decrease in total energy = 19 – 13.74 = 5.3 mJ A1

4 combined capacitance = $(1 / 22 + 1 / 47)^{-1}$ = 14.99 μF C1

initial charge on 22 μF = initial charge on combined capacitors = 14.99 × 12 = 179.88 μC AND charge on 22 μF when p.d. is 6 V = 22 × 6 = 132 μC C1

132 = 179.88 exp[$-t$ / (2.7 × 10^6 × 14.99 × 10^{-6})] C1

Note: For the time constant, it is incorrect to use 22 μF even though we compute the charge using it. This is because the circuit comprises both capacitors and the time constant is due to both capacitors.

t = 12.5 s or 13 s A1

5 **(a)** current in resistor causes capacitor to lose charge, since p.d. proportional to charge so p.d. across capacitor decreases B1

rate of change of p.d. across capacitor decreases as p.d. (or charge) decreases B1

(since $\frac{d}{dt}\left(V_0 e^{-\frac{t}{\tau}}\right) = -\frac{1}{\tau} V_0 e^{-\frac{t}{\tau}}$, gradient of p.d. decreases)

since p.d. across resistor = p.d. across capacitor B1

so rate of change of p.d. across resistor decreases as p.d. (or charge) decreases

(b) [Method 1]
Q_0 = 0.90 mC B1

At t = one time constant, $Q = 0.90\ e^{-1}$ = 0.33 mC M1

Construction on Fig. (b) showing when Q = 0.33 mC, τ = 5.5 s A1

[Method 2]
Evidence of two correct sets of readings for Q and t from the graph B1

Correct substitution of $Q_1 = Q_0 \exp(-t_1 / \tau)$ and $Q_2 = Q_0 \exp(-t_2 / \tau)$ M1
Correct calculation $Q_2 / Q_1 = \exp[(t_1 - t_2) / \tau]$ to give τ = 5.5 s A1

[Method 3]
Read-off of half-life as 3.75 s B1

Use $Q = Q_0 \exp(-t / \tau)$ to show that τ = half-life / ln 2 M1

τ = 3.75 / ln 2 = 5.4 s A1

(c) **(i)** at t = 0, charge on capacitor = 0.90 mC and p.d. = 7.5 V C1

$C = 0.90 \times 10^{-3} / 7.5$ = 120 μF A1

(ii) $R = \tau / C = 5.5 / 120 \times 10^{-6}$ C1

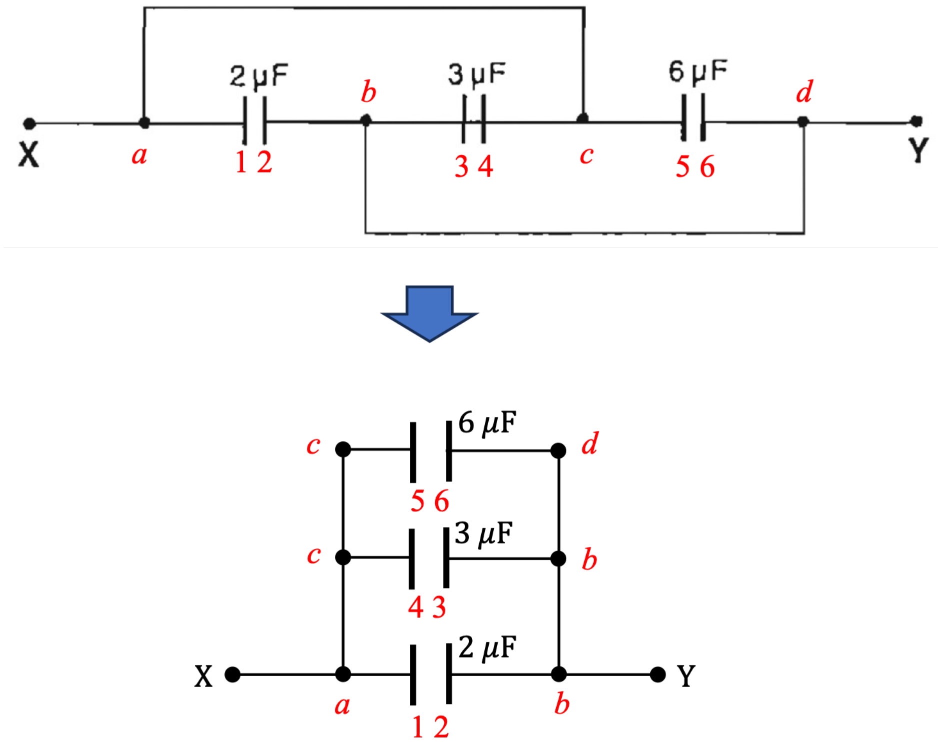
R = 46 kΩ A1

6	**(a)**	V_C equal V_R, so both figures show that Q is proportional to I	M1
		($V_C = Q / C$ and $V_R = IR$, so $Q / C = IR$)	
		current is rate of change of charge	M1
		($I = dQ / dt$)	
		so, Q is proportional to rate of change of Q	A1
		($Q / C = R\, dQ / dt$)	
		this lead to exponential variation of Q	
	(b)	from Fig. (b), $C = 7.2 \times 10^{-3} / 12 = 0.0006$ F	C1
		from Fig. (c), $R = 12 / 1.5 \times 10^{-3} = 8000\ \Omega$	C1
		time constant = $RC = 8000 \times 0.0006 = 4.8$ s	A1

7	(a)	smoothing of output voltage	B1
	(b)	$U = ½\,CV^2$ $0.041 = ½\,C\,(12)^2$	M1
		$C = 569.4 = 570\ \mu F$	A0
	(c)	During discharging, p.d. decreases from 12 V to 8 V in 0.01 s	C1
		$V = V_0 \exp(-t\,/\,RC)$ $8 = 12 \exp[-0.01\,/\,(R \times 570 \times 10^{-6}]$	M1
		$R = 43.27 = 43\ \Omega$	A1

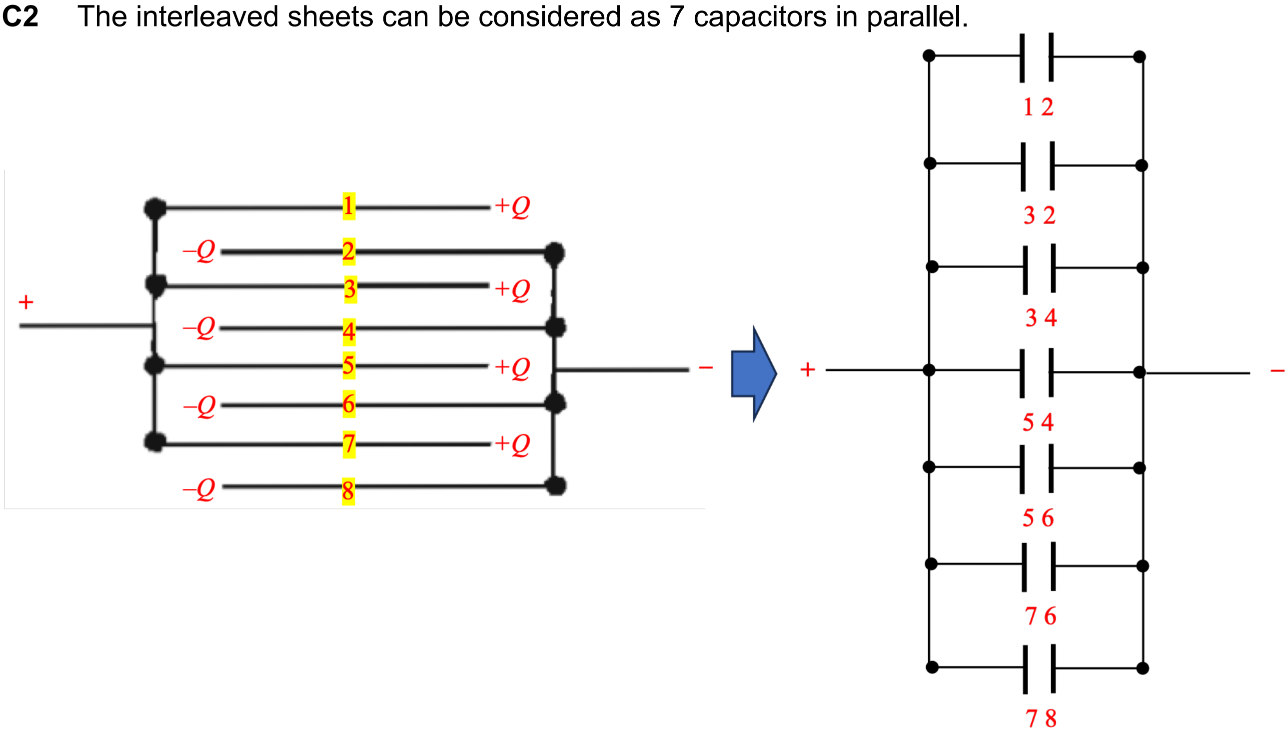
Challenging questions

C1 Rearranging the capacitors systematically,



$C = 2 + 3 + 6 = 11\ \mu\text{F}$

C2 The interleaved sheets can be considered as 7 capacitors in parallel.



Capacitance of each capacitor $C' = \varepsilon_0 \frac{A/4}{d} = \frac{1}{4}\left(\varepsilon_0 \frac{A}{d}\right) = \frac{1}{4}C$

$$C_{\text{eff}} = 7 \times \frac{1}{4}C = \frac{7}{4}C$$

C3 **(a)** Total charge $Q = 2CV_0$

After switch is opened, the capacitors are in parallel. Effective capacitance after adjustment $C_{\text{eff}} = C + fC = C(1+f)$

Steady state p.d. for both capacitors

$$V = \frac{Q}{C} = \frac{2CV_0}{C(1+f)} = \frac{2V_0}{1+f}$$

(b) Before adjustment, energy $U_i = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}(2CV_0)^2\frac{1}{2C} = CV_0^2$

After adjustment

$$U_f = \frac{1}{2}C\left(\frac{2V_0}{1+f}\right)^2 + \frac{1}{2}fC\left(\frac{2V_0}{1+f}\right)^2 = \frac{1}{2}C\left(\frac{2V_0}{1+f}\right)^2(1+f) = \frac{2CV_0^2}{1+f}$$

Difference in energy

$$\Delta U = U_f - U_i = \frac{2CV_0^2}{1+f} - CV_0^2 = \frac{2CV_0^2 - CV_0^2(1+f)}{1+f} = \frac{CV_0^2(1-f)}{1+f}$$

Since $f < 1$, $\Delta U > 0$. Final energy > initial energy.

Work is done to change the variable capacitor value that increases the energy of the system.

C4 **(a)** $E = \frac{1}{2}C_1V_0^2$

(b) **(i)** At steady state, p.d. across both capacitors is V and total charge in the capacitors is the same as the charge in C_1 before switch S_2 is closed.

$C_1V + C_2V = C_1V_0 \Rightarrow V = \frac{C_1}{C_1+C_2}V_0$

Total energy in capacitors at steady state $= \frac{1}{2}C_1V^2 + \frac{1}{2}C_2V^2 \left(= \frac{1}{2}\frac{C_1^2}{C_1+C_2}V_0^2\right)$.

Energy dissipated $E_{\text{diss}} = \frac{1}{2}C_1V_0^2 - \frac{1}{2}\frac{C_1^2}{C_1+C_2}V_0^2$

$\therefore E_{\text{diss}} = \frac{C_1C_2}{2(C_1+C_2)}V_0^2$

(ii) Energy dissipated is total energy of two capacitors at steady state after S_2 is closed, so

$E'_{\text{diss}} = \frac{1}{2}\frac{C_1^2}{C_1+C_2}V_0^2$

C5 **(a)** **(i)** p.d. across network of capacitors is 24 V.

$$C_{\mathrm{WXY}} = \left(\frac{1}{C} + \frac{1}{3C}\right)^{-1} = \frac{3}{4}C$$
$$Q_{\mathrm{WXY}} = \frac{3}{4}C \times 24 = 18C$$
$$V_{\mathrm{WX}} = \frac{Q_{\mathrm{WXY}}}{C} = 18\ \mathrm{V}$$
$$V_{\mathrm{XY}} = \frac{Q_{\mathrm{WXY}}}{3C} = 6\ \mathrm{V}$$

$$C_{\mathrm{WZY}} = \left(\frac{1}{2C} + \frac{1}{4C}\right)^{-1} = \frac{4}{3}C$$
$$Q_{\mathrm{WZY}} = \frac{4}{3}C \times 24 = 32C$$
$$V_{\mathrm{WZ}} = \frac{Q_{\mathrm{WZY}}}{2C} = 16\ \mathrm{V}$$
$$V_{\mathrm{ZY}} = \frac{Q_{\mathrm{WZY}}}{4C} = 8\ \mathrm{V}$$

(ii) $C_{\mathrm{eq}} = \left(\frac{1}{C} + \frac{1}{3C}\right)^{-1} + \left(\frac{1}{2C} + \frac{1}{4C}\right)^{-1} = \frac{25}{12}C$

Therefore, the equivalent capacitance C_{eq} between points W and Y of the network is just over 2C.

(iii) $I = dQ/dt$

$$I = \frac{Q_0}{RC_{\mathrm{eq}}}e^{-\frac{t}{RC_{\mathrm{eq}}}} = \frac{E}{R}e^{-\frac{t}{RC_{\mathrm{eq}}}} = I_0 e^{-\frac{t}{RC_{\mathrm{eq}}}}$$

$$C_{\mathrm{eq}} = \frac{25}{12}C = \frac{25}{12}\left(\frac{500}{4}\right) = \frac{3125}{12}\ \mu\mathrm{F}$$

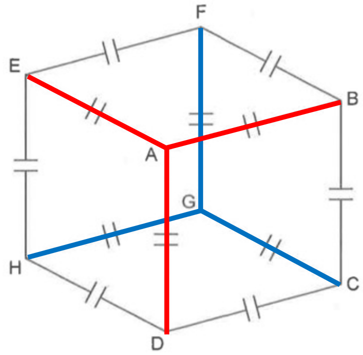
$$\frac{1}{2}I_0 = I_0 e^{-\frac{T}{RC_{\mathrm{eq}}}}$$
$$\ln\frac{1}{2} = -\frac{T}{RC_{\mathrm{eq}}}$$
$$T = RC_{\mathrm{eq}}\ln 2 = (130 \times 10^3)\left(\frac{3125}{12} \times 10^{-6}\right)\ln 2 = 23.5\ \mathrm{s}$$

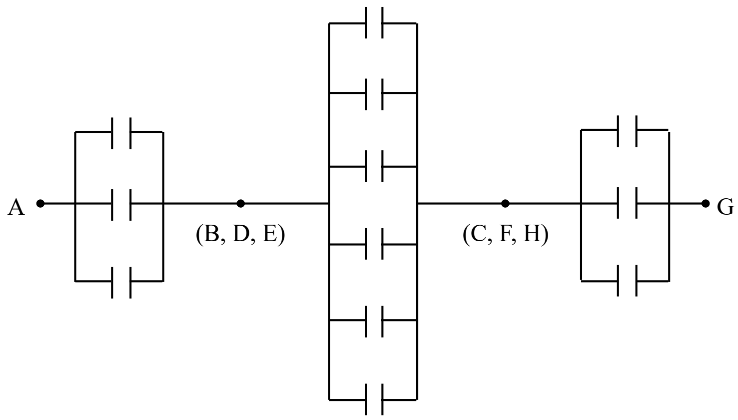
(iv) $C = \frac{500}{4} = 125\ \mu\mathrm{F}$

$Q = CV \Rightarrow q = (125 \times 10^{-6})(18) = 2.25 \times 10^{-3}\ \mathrm{C}$

(v) Energy stored $= \frac{1}{2}C_{\mathrm{eq}}V^2 = \frac{1}{2}\left(\frac{3125}{12} \times 10^{-6}\right)(24)^2 = 0.075\ \mathrm{J}$

(b) Notice the group of 3 capacitors joining to point A and another group of 3 capacitors joining to point G. The other 6 capacitors joins the two groups. The network of capacitors can be redrawn as





(i) BDE or CFH

(ii) $C_T = \left(\frac{1}{3\times25} + \frac{1}{6\times25} + \frac{1}{3\times25}\right)^{-1} = 30\ \mu\text{F}$

C6 **(a)** max charge stored in one cycle $Q = CV$ and period $T = 1/f$ C1

substitution into $I = Q / t$, $2.5 \times 10^{-6} = C \times 180 / (1 / 50)$ M1

$C = 280$ pF A1

(b) total / equivalent capacitance increases M1

greater charge for each cycle / discharge so greater average current A1

C7 **(a)** P labelled "–" and Q labelled "+" B1

(b) V_{OUT} scale labelled 4 and 8 at the tick marks B1

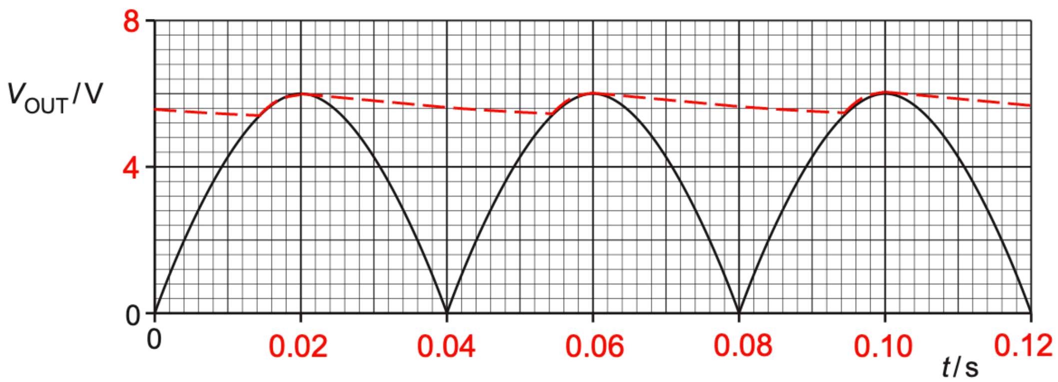
$T = 2\pi / \omega = 2\pi / 25\pi = 0.08$ s B1

t scale labelled 0.02, 0.04, 0.06, 0.08, 0.10, 0.12 on the tick marks B1

(c) **(i)** Correct symbol used for capacitor and capacitor connected in parallel with the 1.2 kΩ resistor. B1

(ii) Straight lines (or curve with negative decreasing gradients) drawn between adjacent peaks, from top of the first peak to meet line going up to the next peak. B1

Line, from one peak to the line going up to the next peak, show a drop in p.d. of 1.5 small squares. B1



(iii) minimum $V_{OUT} = 0.90 \times 6 = 5.4$ V AND discharge time = 0.034 s C1

$V = V_0 \exp(-t / RC)$
$5.4 = 6 \exp[-0.034 / (1.2 \times 10^3 \times C]$ M1

$C = 2.7 \times 10^{-4}$ F A1