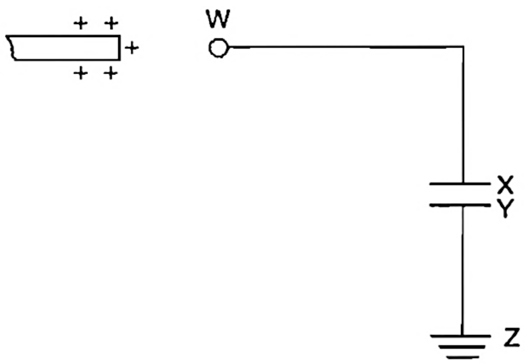
Topic 16: Capacitors
Solutions to Exercises

Exercise 1

1.1 An uncharged capacitor is connected between earth Z and a terminal W. A positively charged rod is brought close to W.



Which of the following describes the movement of charge?

A Electrons move from W to X and from Y to Z.

B Electrons move from W to X but not from Y to Z.

C Electrons move from X to W and from Z to Y.

D Electrons move from X to W but not from Z to Y.

Answer: C
Electrons near W attracted to positive rod causing electrons to move towards W.
X becomes net positive.
X being positive attracts electrons to move towards Y, drawing electrons from the earth.

1.2 A 20 μF capacitor is charged by a constant current of 10 mA. If the capacitor is initially uncharged, how long does it take for the potential difference across the capacitor to reach 300 V?

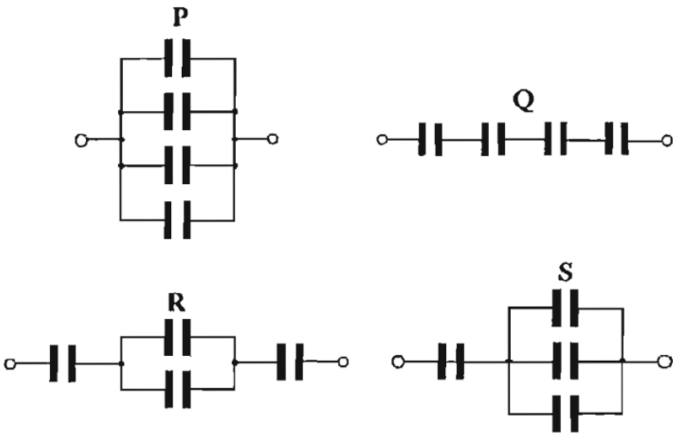
A 6.0×10^{-4} s **B** 0.60 s **C** 15 s **D** 6.0×10^{5} s

Answer: B
Charge on capacitor when p.d. is 300 V, $Q = CV = (20 \times 10^{-6})(300) = 0.006$ C
Since current = charge / time,

$10 \times 10^{-3} = 0.006 / t$

$t = 0.6$ s

1.3 Four identical capacitors are connected as shown.

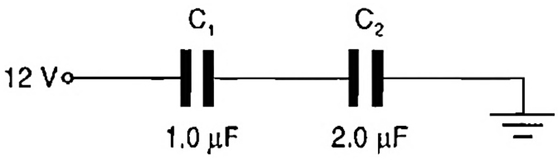


Which of the following lists the arrangements in order of **decreasing** capacitance?

A PQRS **B** PSRQ **C** QRSP **D** QSRP

Answer: B
Let C be the capacitance of each capacitor.
Circuit P: $C_{\text{total}} = C + C + C + C = 4C$
Circuit Q: $C_{\text{total}} = [1/C + 1/C + 1/C + 1/C]^{-1} = C/4$
Circuit R: $C_{\text{total}} = [1/C + 1/(C + C) + 1/C]^{-1} = 2C/5$
Circuit S: $C_{\text{total}} = [1/C + 1/(C + C + C)]^{-1} = 3C/4$
Decreasing capacitance is PSRQ.

1.4 Two capacitors are connected in series as shown.



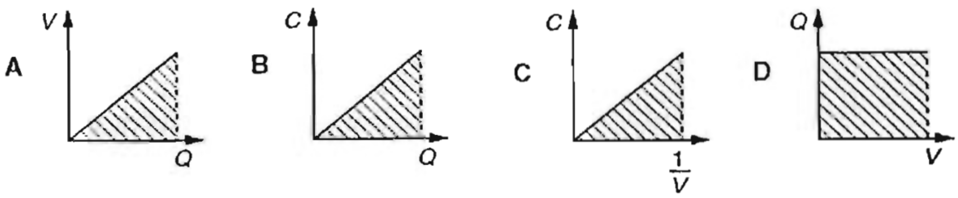
What is the charge carries by each of these capacitors?

	charge on C_1 / μC	charge on C_2 / μC
A	4.0	4.0
B	4.0	8.0
C	8.0	4.0
D	8.0	8.0

Answer: D
$C_{\text{total}} = (1/1.0 + 1/2.0)^{-1} = 2/3$ μF
p.d. across equivalent capacitor $V = 12 – 0 = 12$ V
Charge on equivalent capacitor = $C_{\text{total}}\, V = (2/3)\,(12) = 8$ μC
Capacitors in series have the same amount of charge.

1.5 The energy stored in a capacitor of capacitance *C*, carrying charge *Q* with potential difference *V* between its plates. may be obtained by calculating the area under an appropriate graph.

Which graph shows the correct relationship between a pair of the quantities *C*, *Q* and *V,* and in addition shows a shaded area which corresponds to the energy stored in the capacitor?



Answer: A

C = *Q* / *V* and energy stored in capacitor = $\frac{1}{2}QV = \frac{1}{2}CV^2 = \frac{1}{2}\frac{Q^2}{C}$. So

Option A: Graph of *V* against *Q* gives a straight line with gradient = 1/*C*.
Area under graph = $\frac{1}{2}QV$ = energy stored

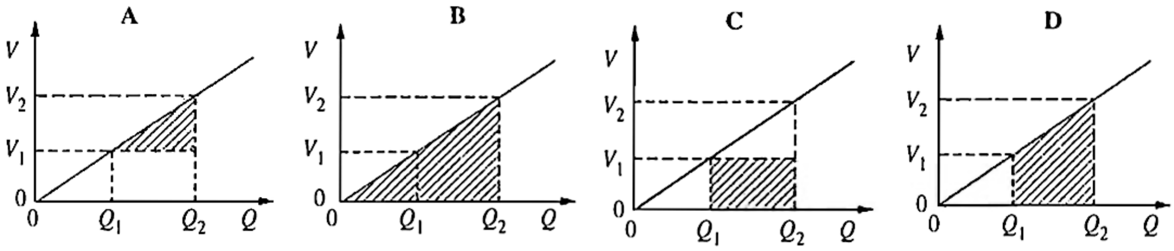
Option B: Graph of *C* against *Q* gives a straight line with gradient = 1/*V*.
Area under graph = $\frac{1}{2}QC \neq$ energy stored

Option C: Graph of *C* against 1/*V* gives a straight line with gradient = *Q*.
Area under graph = $\frac{1}{2}C\left(\frac{1}{V}\right) = \frac{1}{2}\frac{C}{V} \neq$ energy stored

Option D: Graph of *Q* against *V* cannot be a horizontal line.

1.6 When the potential difference *V* between the plates of a capacitor is increased from V_1 to V_2, the charge *Q* on the plates increases from Q_1 to Q_2.

On the graphs, which shaded area represents the increase in the energy stored in the capacitor?



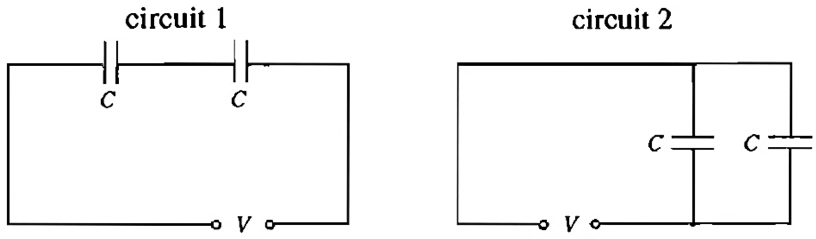
Answer: D

Initial energy stored = ½ Q_1V_1

Final energy stored = ½ Q_2V_2

Increase in energy stored = ½ Q_2V_2 – ½ Q_1V_1

1.7 The diagrams show two ways of connecting two identical capacitors of capacitance *C* in circuits each with a supply of p.d. *V*.



What is the value of $\dfrac{\text{total energy stored in the capacitors in circuit 1}}{\text{total energy stored in the capacitors in circuit 2}}$?

A 0.25 **B** 0.50 **C** 2.0 **D** 4.0

Answer: A

Circuit 1: Equivalent capacitor $C_{total} = (1/C + 1/C)^{-1} = C / 2$
p.d. across equivalent capacitor = V
Energy stored = ½ $(C / 2)\ V^2$ = ¼ CV^2

Circuit 2: Equivalent capacitor $C_{total} = C + C = 2C$
p.d. across equivalent capacitor = V
Energy stored = ½ $(2C)\ V^2 = CV^2$

Ratio = ¼ / 1 = 0.25

1.8 A capacitor is charged to a voltage *V* and then discharged through a small d.c. motor. As the capacitor discharges, the motor raises a mass through a height *h*. The experiment is repeated for several values of *V*.

A constant fraction of the capacitor energy is converted to gain of gravitational potential energy.

Which graph would be expected to give a straight line?

A h against V^2 **B** h against V

C h against $\sqrt{V}$ **D** h against $1/\sqrt{V}$

Answer: A

Energy stored in capacitor $U = \frac{1}{2}CV^2$

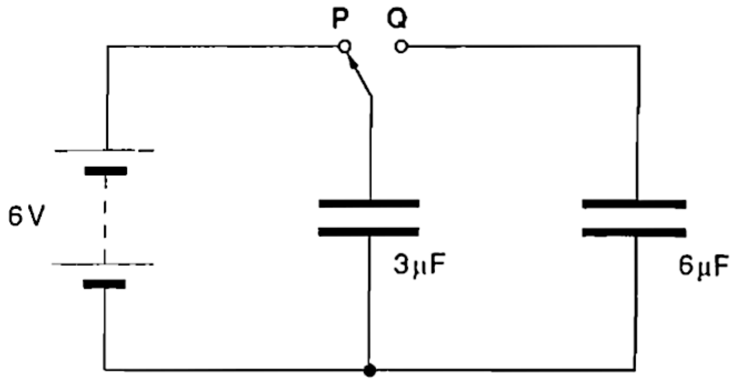
Energy transferred to gravitational potential store = $k \times U = mgh$ where k is a constant < 1.

$$k \times \frac{1}{2}CV^2 = mgh$$

$$h = \frac{kC}{2mg}V^2$$

Since k, C, m and g are constant, a graph of h against V^2 will give a straight line.

1.9 In the circuit shown, a capacitor of capacitance 3 μF is charged from a battery of e.m.f. 6 V with the switch connected to terminal **P**.



The switch is now connected to **Q**. This charges the 6 μF capacitor from the 3 μF one.

What is the new potential difference across the combination?

A 1 V **B** 2 V **C** 4 V **D** 6 V

Answer: B

Before connecting to Q, charge on 3 μF capacitor = (3)(6) = 18 μC

After connecting to Q,

equivalent capacitance of parallel connected capacitors = 3 + 6 = 9 μF

charge on equivalent capacitor = 18 μC

p.d. across equivalent capacitor $V = Q / C$ = 18 / 9 = 2 V

1.10 A capacitor of capacitance C_1 is charged to a potential difference of 100 V and then disconnected. When a second uncharged capacitor of capacitance C_2 is connected in parallel across it, the new p.d. is 60 V.

What is the value of the ratio $\dfrac{C_1}{C_2}$?

A 0.60 **B** 0.67 **C** 1.50 **D** 1.67

Answer: C

Before connecting to C_2 capacitor, charge on C_1 capacitor = $100C_1$

After connecting to C_2 capacitor,

equivalent capacitance of parallel connected capacitors = $C_1 + C_2$

charge on equivalent capacitor = $100C_1$

p.d. across equivalent capacitor $V = Q / C = 100C_1 / (C_1 + C_2) = 60$ V

$$\frac{C_1 + C_2}{100C_1} = \frac{1}{60}$$

$$1 + \frac{C_2}{C_1} = \frac{100}{60}$$

$$\frac{C_1}{C_2} = \frac{60}{40} = 1.50$$