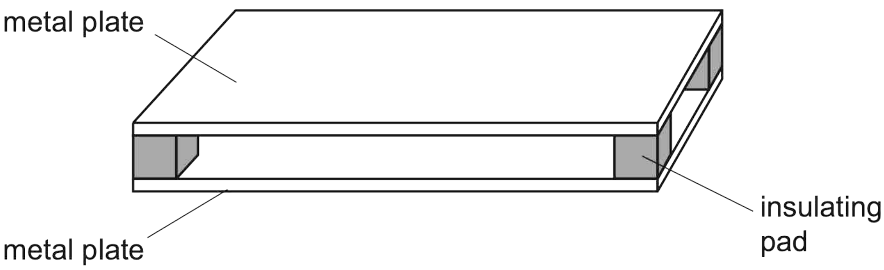
16. Capacitors
Problem set

Formulae	
capacitors in series	$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots$
capacitors in parallel	$C = C_1 + C_2 + \cdots$
energy in a capacitor	$U = \frac{1}{2}QV = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}CV^2$
charging a capacitor	$Q = Q_0\left[1 - e^{-\frac{t}{\tau}}\right]$
discharging a capacitor	$Q = Q_0 e^{-\frac{t}{\tau}}$
RC time constant	$\tau = RC$

(Suggested duration: 120 minutes)

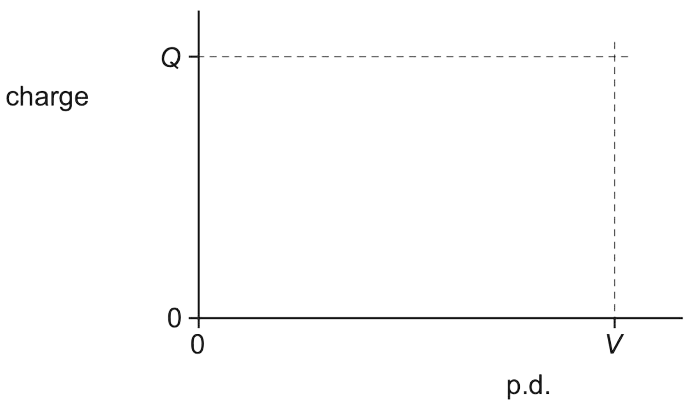
1 **(a)** Two flat metal plates are held by a small distance apparat by means of insulating pads.



Explain how the plates could act as a capacitor. [2]

(b) An initially uncharged capacitor X, of capacitance *C*, is gradually charged by a power supply so that the final potential difference (p.d.) between its plates is *V* and the final charge is *Q*.

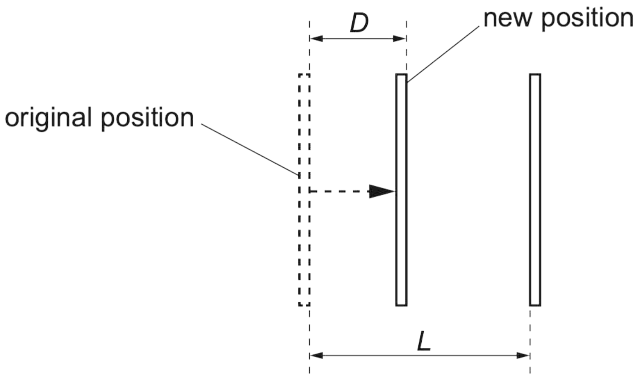
(i) Sketch the variation of charge with p.d. for capacitor X as the p.d. increases from 0 to *V*.



[2]

(ii) Determine an expression, in terms of *Q* and *V*, for the work *W* done on capacitor X during the charging process. Explain your reasoning. [2]

(c) The charged capacitor in **(b)** is now disconnected from the supply. The plates of the capacitor are initially separated by distance L. They are then moved closer together by a distance D as shown.



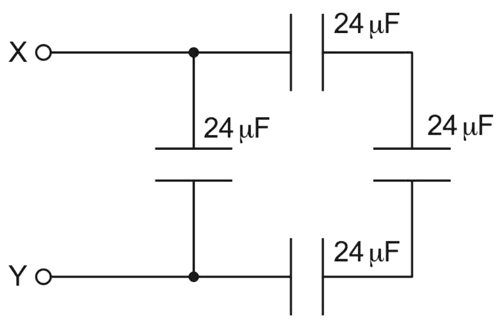
(i) By considering the electric field between the plates, show that the new p.d. V_N between the plates is given by:

$$V_N = \left(\frac{L-D}{L}\right)V$$

Explain your reasoning. [2]

(ii) Deduce the expression for the new capacitance C_N in terms of C, L and D. Explain your reasoning. [3]

2 **(a)** A student has available four capacitors, each of capacitance 24 μF.



The capacitors are connected as shown.

Calculate the combined capacitance between the terminals X and Y. [2]

(b) A student has available **three** capacitors, each of capacitance 12 μF.

Draw diagrams, one in each case, to show how the student connects the capacitors to give a combined capacitance between the terminals of:

(i) 18 μF, [1]

(ii) 8 μF. [1]

3 **(a)** Two parallel plate capacitors C_1 and C_2 are connected to a supply that has a potential difference (p.d.) V_S. The capacitors may be connected in series or in parallel.

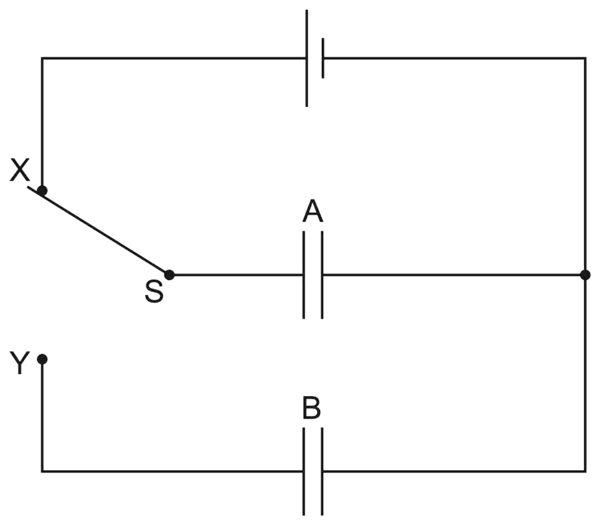
The supply provides charge Q_S and the plates of the two capacitors acquire charges Q_1 and Q_2 respectively. The p.d.s across the plates of the capacitors are V_1 and V_2 respectively.

Complete the table below to indicate how Q_S, Q_1 and Q_2 relate to each other, and how V_S, V_1 and V_2 relate to each other, for series and parallel connections of the capacitors to the supply.

	relationship between charges	relationship between p.d.s
series		
parallel		

[4]

(b) Two capacitors A and B are connected into the circuit shown below.



Capacitor A has capacitance 470 μF and capacitor B has capacitance 180 μF.
The two-way switch S is initially at position X, and capacitor B is initially uncharged.
Capacitor A stores 19 mJ of energy when fully charge.

(i) Calculate the e.m.f. of the cell. [2]

(ii) Calculate the charge on the capacitor A. [2]

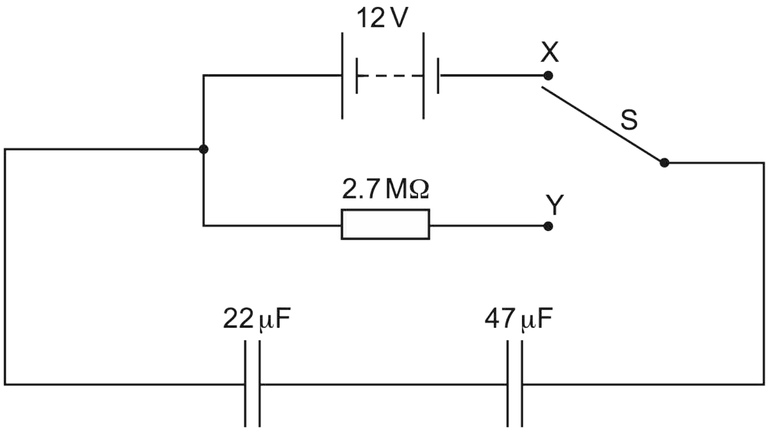
(iii) The two-way switch S is now moved to position Y.

Determine

1. the final p.d. across capacitor B, [3]

2. the decrease in the total energy, in mJ, stored in the two capacitors. [2]

4 Two capacitors of capacitances 22 μF and 47 μF, and a resistance of 2.7 MΩ, are connected into the circuit as shown.



The battery has an e.m.f. of 12 V.
The two-way switch S is initially at position X, so that the capacitors are fully charged.

The two-way switch is then moved to position Y.
Determine the time taken for the p.d. across the 22 μF capacitor to become 6.0 V. [4]

5 A capacitor of capacitance C and a resistor of resistance R are connected, as shown in Fig. (a).

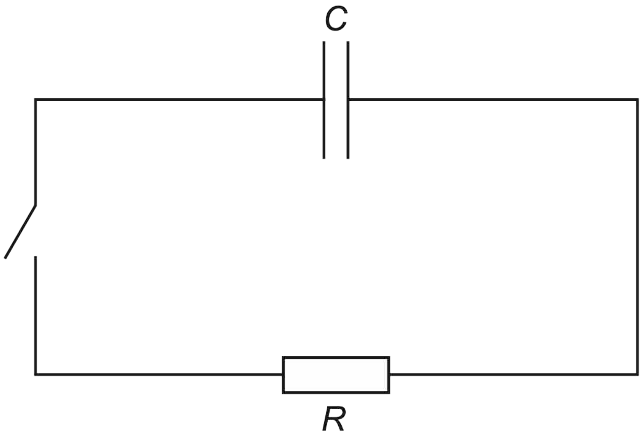


Fig. (a)

Initially, the capacitor is charged and the switch is open.

The switch is closed at time $t = 0$.

Fig. (b) and Fig. (c) show, respectively, the variations with t of the charge Q on the capacitor and the potential difference (p.d.) V across the resistor.

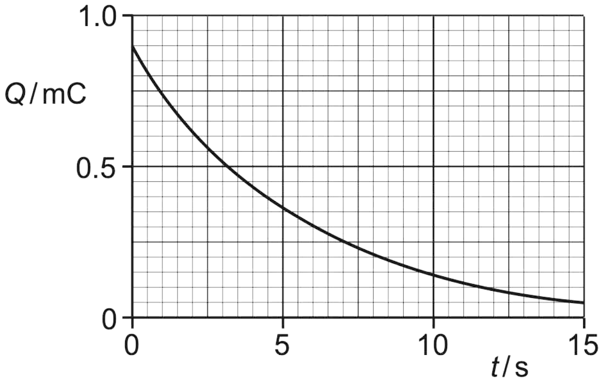


Fig. (b)

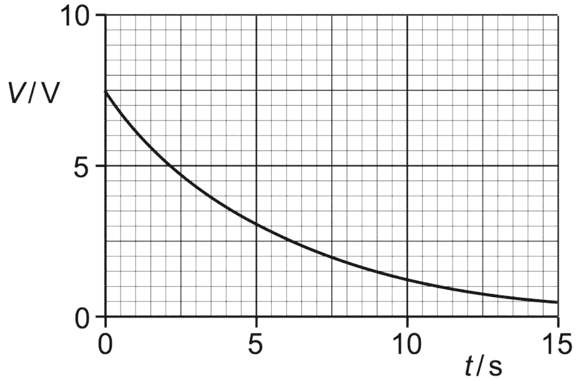


Fig. (c)

(a) Explain the shape of the line in Fig. (c) representing the variation of V with t. [3]

(b) Use Fig. (b) to show that the time constant of the circuit in Fig. (a) is 5.5 s. [3]

(c) Using Fig. (b), Fig. (c) and the information in **(b)** to determine:

(i) capacitance C, in μF, [2]

(ii) resistance R, in kΩ. [2]

6 Fig. (a) shows a circuit contains a capacitor of capacitance C and a resistor of resistance R.

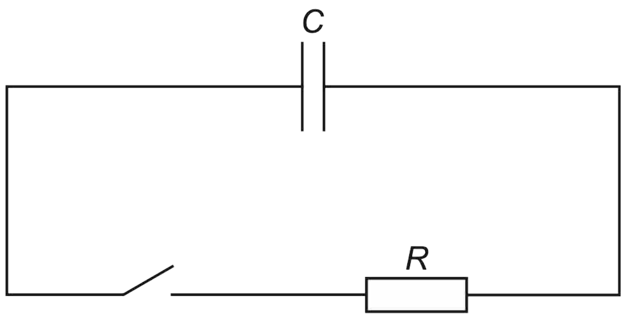


Fig. (a)

Initially, the switch is open and the p.d. across the capacitor is 12 V.

The switch is closed at time t = 0 and the capacitor discharges through the resistor.

Fig. (b) shows the variation of the charge Q on the capacitor with the p.d. V_C across the capacitor as the capacitor discharges. Fig. (c) shows the variation of the current I in the resistor with the p.d. V_R across the resistor as the capacitor discharges.

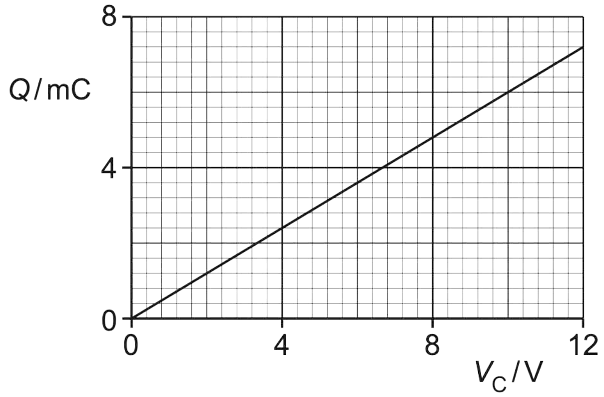


Fig. (b)

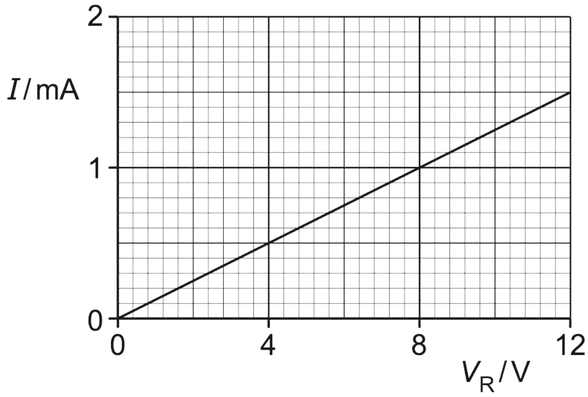


Fig. (c)

Use Fig. (b) and Fig. (c) to

(a) explain why the variation of Q with t is exponential in nature. [3]

(b) determine the time constant of the circuit. [3]

7 The circuit in Fig. (a) produces half-wave rectification of an alternating input voltage V_{IN} to produce output voltage V_{OUT} across the resistor R.

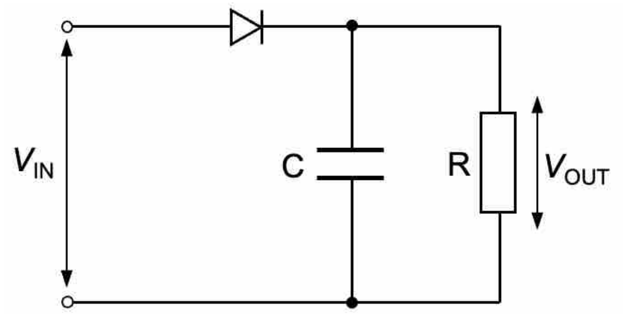


Fig. (a)

The input voltage V_{IN} is a square wave. Fig. (b) shows the variation of V_{IN} with time t.

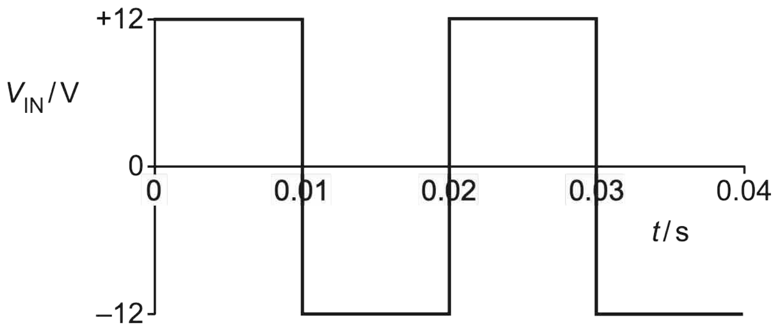


Fig. (b)

Fig. (c) shows the variation of the output voltage V_{OUT} with t.

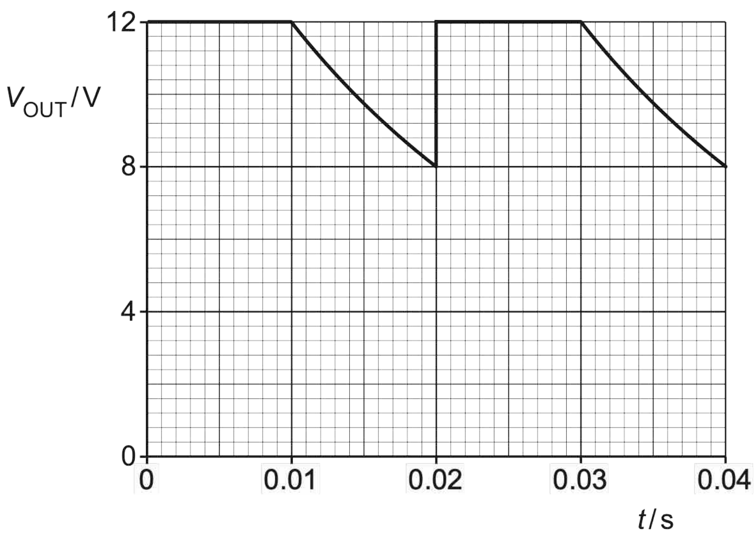


Fig. (c)

The maximum energy stored in the capacitor is 0.041 J.

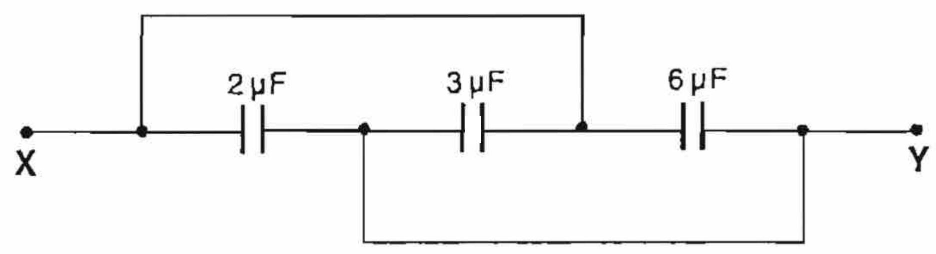
(a) State the purpose of the capacitor C in the circuit. [1]

(b) Show that the capacitance of C is 570 μF. [1]

(c) Determine the resistance of R. [3]

Challenging questions

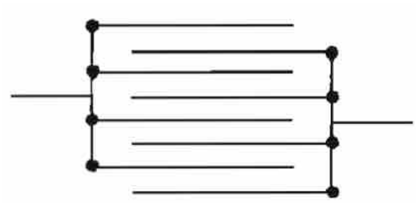
C1 Three capacitors are arranged as shown.



Determine the effective capacitance between X and Y.

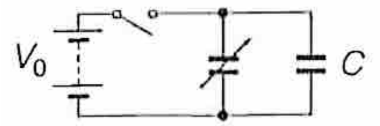
C2 A capacitor made from two thin, flat metal sheets separated by a small thickness of insulating material has a capacitance *C*.

Each metal sheet is then cut into four smaller identical sheets, which are used to make another capacitor as shown. The same thickness of insulator is used between the interleaved sheets.



Neglecting end-effects, determine the capacitance of the reconstructed capacitor.

C3 A circuit containing a capacitor of fixed capacitance *C* and a variable capacitor. The e.m.f. of the battery is V_0. Initially, with the switch closed, the variable capacitor is set so as to have a capacitance of exactly *C*. The switch is then opened. Finally, with the switch still open, the variable capacitor is adjusted to have a capacitance of *fC*, where *f* is less than 1.



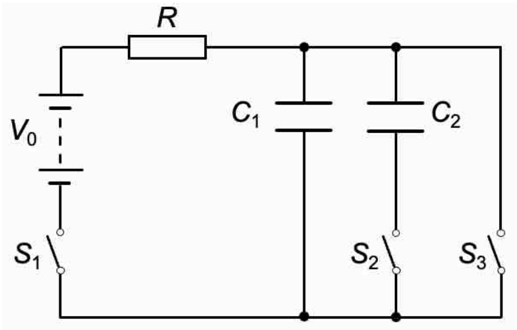
(a) Show that the resulting potential difference V_c across the fixed capacitor is given by

$$V_c = \frac{2V_0}{1+f}$$

and determine the potential difference across the variable capacitor.

(b) Show that the total energy stored in the capacitors after the final adjustment is more than the initial total energy, and find an expression for the energy difference. Give your answer in terms of *f*, *C* and V_0. Explain how you can reconcile this result with the principle of conservation of energy.

C4 In the circuit shown, all three switches S_1, S_2 and S_3 are initially open. The battery has an e.m.f. V_0 with no internal resistance. The capacitors C_1 and C_2 are ideal. The connecting wires have zero resistance.



(a) While switches S_2 and S_3 are open, state the expression for the energy E stored in the capacitor C_1 in terms of V_0, R and C_1. [1]

(b) After a long time, the capacitor C_1 becomes fully charged. Switch S_1 is then opened.

(i) On closing switch S_2, while S_1 and S_3 remain open, it is noticed that a spark develops between the contacts of the switch.

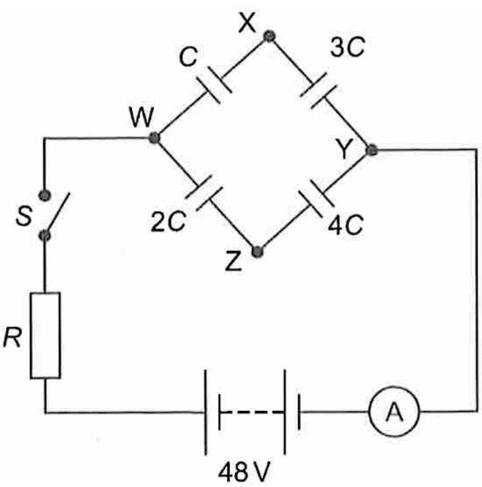
By considering the change in energy stored in the capacitors, show that the energy dissipated E_{diss} in this spark is:

$$E_{diss} = \frac{C_1 C_2}{2(C_1 + C_2)} V_0^2$$

(ii) Switch S_3 is now closed.

State the expression for the energy E'_{diss} dissipated in the spark which develops across it in terms of V_0, R, C_1 and C_2.

C5 A network of four capacitors of capacitance C, $2C$, $3C$ and $4C$ is constructed and placed into an electrical circuit as shown.



The capacitors are initially uncharged and switch S is open.

At time t = 0, switch S is closed and an initial current I_0 is shown on the ammeter.

At time t = T, the current shown on the ammeter is $\frac{I_0}{2}$ and the capacitor of capacitance C stores a charge of magnitude q on each plate.

(a) **(i)** Determine the potential difference across each of the capacitors at time t = T.

(ii) Show that the equivalent capacitance C_{eq} between points W and Y of the network is just over 2*C*.

(iii) The capacitance 4*C* is 500 μF. The resistor has a resistance *R* = 130 kΩ.

The total charge Q on the capacitor network at time *t* is given by the equation:

$$Q = Q_0\left(1 - e^{-\frac{t}{RC_{\mathrm{eq}}}}\right)$$

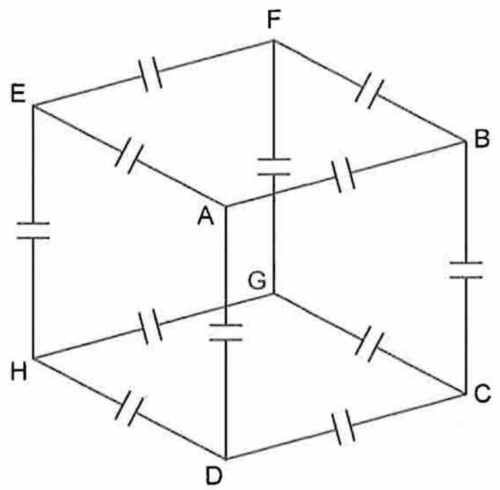
where Q_0 is the maximum charge on the capacitor network.

By differentiating the equation of the total charge Q with respect to time, calculate the value of the time *T*.

(iv) Use your answer to **(a)(i)** and information from **(a)(iii)** to determine the value of *q*.

(v) Determine the energy stored in the capacitor network at time *t* = *T*.

(b) Twelve identical capacitors each of capacitance 25 μF are connected so that they form the sides of a cube as shown.

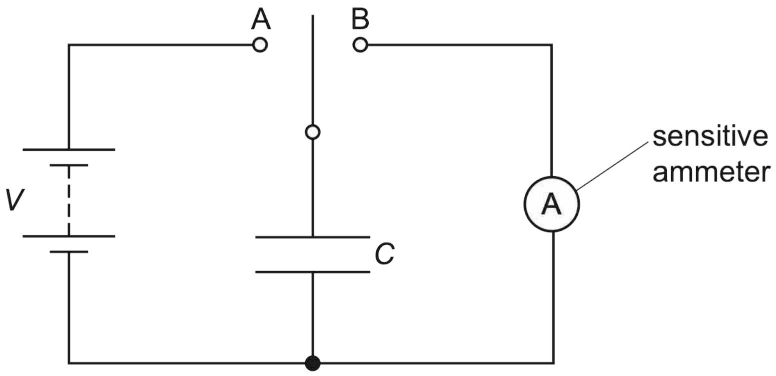


A potential difference is applied between A and G.

(i) State the letters of three points in the circuit that have the same potential. [1]

(ii) Calculate the total capacitance C_{T} of the cube of capacitors. [2]

C6 A capacitor of capacitance C is connected into a circuit shown below.



When a two way switch is moved to position A, the capacitor is charged so that the potential difference across it is V. When the switch moves to position B, the capacitor fully discharges through the sensitive ammeter.

The switch moves repeatedly between A and B so that the capacitor charges and then discharges with frequency f.

(a) For a potential difference V of 180 V and a frequency f of switching of 50 Hz, the average current I in the ammeter is 2.5 μA.

Calculate the capacitance, in pF, of the capacitor. [3]

(b) A second capacitor is connected into the circuit. The two capacitors are connected in parallel.

State and explain the change, if any, in the average current in the ammeter. [2]

C7 Four diodes are needed to produce a full-wave rectification of an alternating input voltage V_{IN} to produce output voltage V_{OUT} across the resistor of resistance 1.2 kΩ using the circuit shown in Fig. (a).

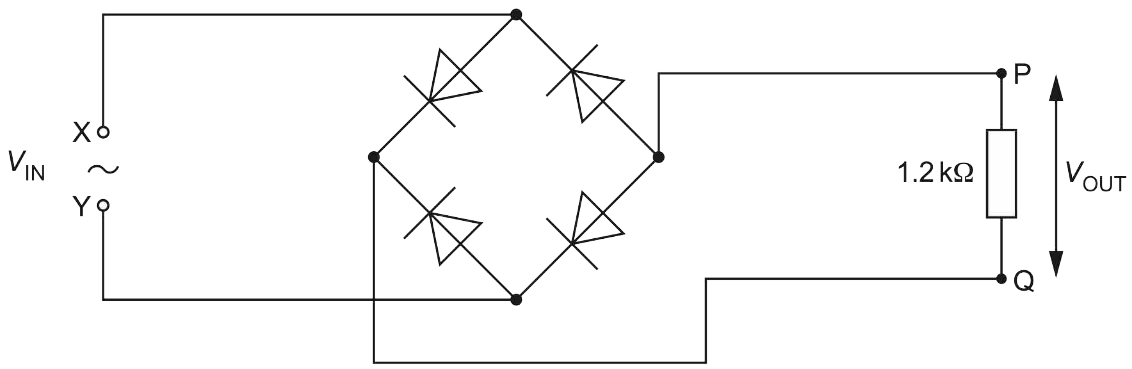


Fig. (a)

A sinusoidal alternating voltage V_{IN} is applied across the input terminals X and Y. The variation with time t of V_{IN} is given by the equation

$$V_{IN} = 6.0 \sin 25\pi t$$

where V_{IN} is in volts and t is in seconds.

(a) On Fig. (a), label the output terminals P and Q with the appropriate symbols to indicate the polarity of the output voltage V_{OUT}.

(b) The magnitude of the output voltage V_{OUT} varies with t as shown in Fig. (b).

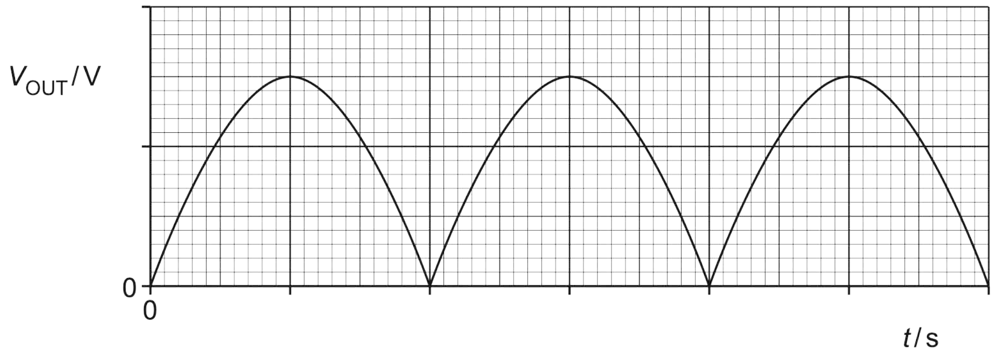


Fig. (b)

On Fig. (b), label both of the axes with the correct scales. Show any working that you need. [3]

(c) The output voltage in **(b)(ii)** is smoothed by adding a capacitor to the circuit in Fig. (a). The difference between the maximum and minimum values of the smoothed output voltage is 10% of the peak voltage.

(i) On Fig. (a), draw the circuit symbol for a capacitor showing the capacitor correctly connected into the circuit. [1]

(ii) On Fig. (b), sketch the variation with t of the smoothed output voltage. [1]

(iii) Calculate the capacitance of the capacitor. [3]

Numerical answers

1 **(b)(i)** $W = \frac{1}{2} QV$ **(c)(ii)** $C_N = CL / (L - D)$

2 **(a)** 32 μF

3 **(b)(i)** 9.0 V **(ii)** 4.2×10^{-3} C **(iii)** 14 mJ

4 13 s

5 **(c)(i)** 120 μF **(ii)** 46 kΩ

6 **(b)** 4.8 s

7 **(c)** 43 Ω

C1 11 μF

C2 $7C / 4$

C3 **(b)** $\frac{CV_0^2(1-f)}{1+f}$

C4 **(a)** $E = \frac{1}{2}C_1 V_0^2$ **(b)(ii)** $E'_{\text{diss}} = \frac{1}{2}\frac{C_1^2}{C_1+C_2}V_0^2$

C5 **(a)(i)** $V_{WX} = 18$ V, $V_{XY} = 6$ V, $V_{WZ} = 16$ V, $V_{ZY} = 8$ V **(iii)** 23.5 s

(iv) 2.25 mC **(v)** 0.075 J

(b)(i) BDE or CFH **(ii)** 30 μF

C6 **(a)** 280 pF

C7 **(c)(iii)** 2.7×10^{-4} F