3 MOTION & FORCES

H2 Physics 9478

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Learning Outcomes

Candidates should be able to:

(a) show an understanding of and use the terms position, distance, displacement, speed, velocity, and acceleration.

(b) use graphical methods to represent distance, displacement, speed, velocity, and acceleration.

(c) identify and use the physical quantities from the gradients of position-time or displacement-time graphs, and areas under and gradients of velocity-time graphs, including cases of non-uniform acceleration.

(d) derive, from the definitions of velocity and acceleration, equations which represent uniformly accelerated motion in a straight line.

(e) solve problems using equations which represent uniformly accelerated motion in a straight line, e.g. for bodies falling vertically without air resistance in a uniform gravitational field.

(f) show an understanding that mass is the property of a body which resists change in motion (inertia).

(g) define and use linear momentum as the product of mass and velocity.

(h) state and apply each of Newton's laws of motion:

1st law: a body at rest will stay at rest, and a body in motion will continue to move at constant velocity, unless acted on by a resultant external force;

2nd law: the rate of change of momentum of a body is (directly) proportional to the resultant force acting on the body and is in the same direction as the resultant force; and

3rd law: the force exerted by one body on a second body is equal in magnitude and opposite in direction to the force simultaneously exerted by the second body on the first body.

(i) recall the relationship resultant force $F = ma$ for a body of constant mass, and use this to solve problems.

3.1 Kinematics

❖ Introduction

The study of the motion of objects, with the associated concepts of force and energy, is called *mechanics*.

Mechanics can be further divided into two parts:

1. *kinematics* which describe how objects move and
2. *dynamics* which deal with force and why objects move as they do.

Motion can be categorized into three types:

- translational,
- rotational and
- vibrational.

In this chapter, we are concerned only with translational motion and will treat the moving object as a *particle,* regardless of its size.

Strictly speaking, a particle is a point-like object with mass but no size. However, we can still apply the particle model to objects such as a ball or a car, provided the positions of the objects refer to their centres of mass.

We begin our study with *rectilinear motion*, which is motion in one dimension or motion in a straight line, and then proceed to projectile motion, which is an example of motion in a two-dimensional plane.

Circular motion is another example of motion in a plane and that will be covered in a later chapter.

All measurements of distance or speed are made relative to a frame of reference. Unless otherwise specified, the frame of reference is assumed to be that of a stationary observer on the Earth. Once the frame of reference is specified, it is then represented by a coordinate system.

For motion in a plane, a pair of mutually perpendicular axes is chosen. The most common pair of axes is the horizontal and vertical axes, usually labelled as the *x*- and *y*- axes respectively.

The point of launch or start of motion is usually designated as the origin and the directions for the *x*- and *y*- axes are chosen arbitrarily from the origin.

❖ Defining the Basics

Scalar	Vector
Distance, *x*	**Displacement, *s***
The total length of path an object travels. SI unit is *metre* (m).	The distance moved in a specified direction from a reference point. SI unit is *metre* (m).

Speed, *v*	**Velocity, *v***
The **instantaneous speed** of an object is defined as the rate of change of <u>distance</u> travelled with respect to time. $v = \frac{dx}{dt}$ SI unit is *metre per second* (m s^{-1}).	The instantaneous velocity of an object is defined as the rate of change of displacement with respect to time. $v = \frac{ds}{dt}$ SI unit is *metre per second* (m s^{-1}).
Average speed refers to the total distance travelled over total time taken. $\langle v \rangle = \frac{\Delta x}{\Delta t}$	**Average velocity** refers to the change in displacement over total time taken. $\langle v \rangle = \frac{\Delta s}{\Delta t}$

	Acceleration, *a*
No scalar equivalent of acceleration.	The **instantaneous acceleration** of an object is defined as the rate of change of velocity with respect to time. $a = \frac{dv}{dt}$ SI unit is *metre per second squared* (m s^{-2}).
	Average acceleration refers to the change in velocity over time taken. $\langle a \rangle = \frac{\Delta v}{\Delta t}$

❖ Graphs in Kinematics

Graphs are very useful in representing the changes that occur during the motion of an object. There are three possible graphs that can provide useful information:

- displacement-time (s-t) graph
- velocity-time (v-t) graph
- acceleration-time (a-t) graph

Information from the graphs is often obtained from

1. direct reading of a point on the line / curve,
2. the gradient of the graph, and
3. the area under the graph.

The physical quantities obtained depend on what is being plotted on the graph. **Always look at the axes of a graph very carefully.**

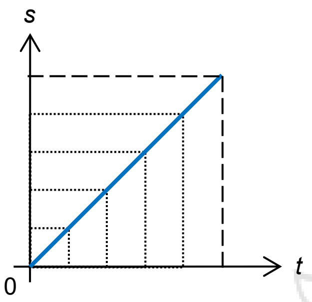
	***s-t* graph**	***v-t* graph**	***a-t* graph**
Gradient at a point	$\frac{ds}{dt}$ instantaneous velocity	$\frac{dv}{dt}$ instantaneous acceleration	no physical significance
Area under graph	no physical significance	$\int v \; dt = \Delta s$ = *change* in displacement	$\int a \; dt = \Delta v$ = *change* in velocity

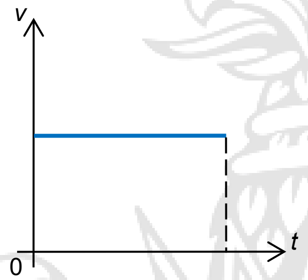
❖ Conversion between graphs

In order to analyze the motion of objects fully, it is important to be able to convert the graphs from one form to another.

Case 1: Constant velocity

Graphs for an object moving with constant velocity.





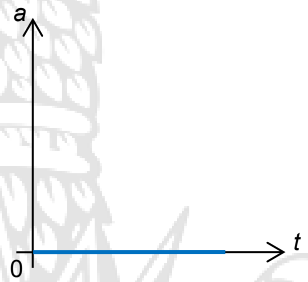
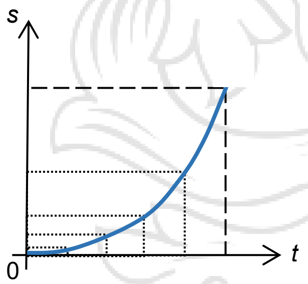
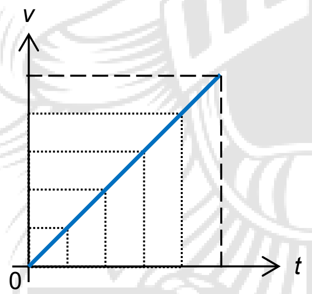


Fig. 3.1(a) **Fig. 3.1(b)** **Fig. 3.1(c)**

Case 2: Increasing velocity under constant acceleration

Graphs for an object moving with constant acceleration.





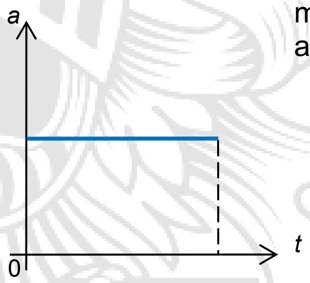


Fig. 3.2(a) **Fig. 3.2(b)** **Fig. 3.2(c)**

3.2 Uniformly accelerated linear motion

❖ Kinematics equations

The motion of an object whose velocity is increasing at a steady rate is called **uniformly accelerated motion**. The graph below shows the velocity-time graph of an object moving with constant acceleration. Its initial velocity is u and its velocity at time t later is v.

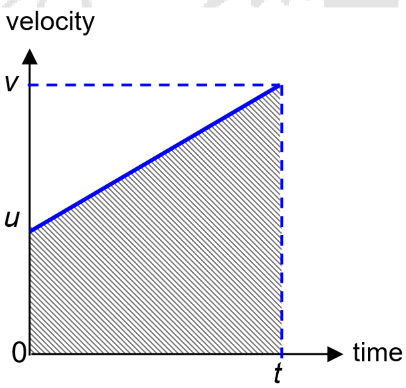


Fig. 3.3

Acceleration a = gradient of v-t graph

$$a = \frac{v-u}{t}$$

Hence,

$$\boxed{v = u + at} \quad \ldots\ldots\ldots\ldots \textbf{(1)}$$

Displacement s = Area under v-t graph

$$\boxed{s = \frac{1}{2}(u+v)t} \quad \ldots\ldots\textbf{(2)}$$

These 4 equations apply only to motion in a <u>straight line</u> with <u>constant acceleration</u>.

Substituting (1) into (2),

$$s = \frac{1}{2}(u + (u + at))t$$

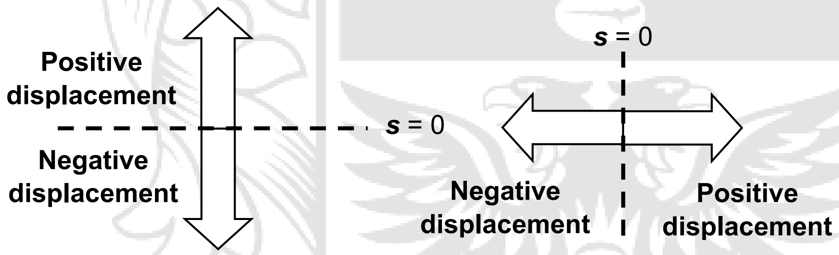
$$\boxed{s = ut + \frac{1}{2}at^2} \quad \ldots\ldots\ldots\textbf{(3)}$$

From (1), $t = \frac{v-u}{a}$ and substituting into (2),

$$s = \frac{1}{2}(u+v)\left(\frac{v-u}{a}\right) = \frac{v^2-u^2}{2a}$$

$$\boxed{v^2 = u^2 + 2as} \quad \ldots\ldots\ldots\textbf{(4)}$$

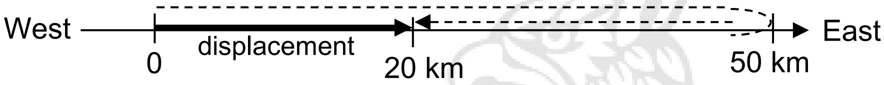
Before solving any problem involving vector quantities (e.g. s, v, a), there is a need to **define the positive direction**. Thereafter, all the quantities will take reference to this defined direction for their sign conventions.



However, it is possible to define the positive direction opposite to the convention mentioned above whenever it is more convenient to do so.

Example 1

Consider a car moving 50 km from West to East and then 30 km from East to West.



Example 2

A car travels 5.0 km due East from a point O to a point A and then a further 2.0 km southward. What is the distance covered and the magnitude of its displacement?

Example 3

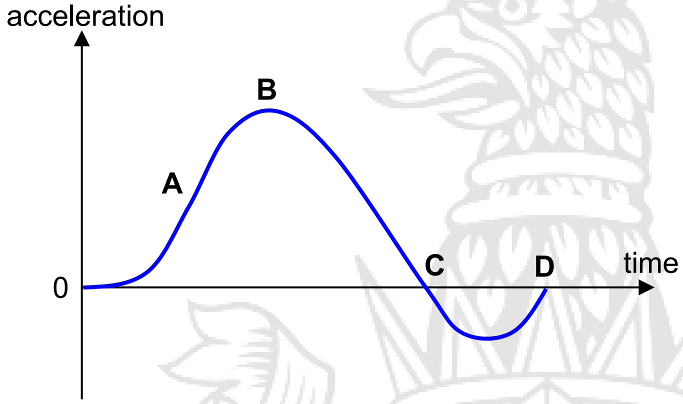
An athlete runs at a steady speed clockwise round a running track of perimeter 400 m and completes one round in 1 min 20 s.

Calculate
(a) her average speed for one round,
(b) her average velocity for one round.

Example 4

The graph below shows the acceleration-time graph of an object moving from rest. At which points on the graph does the object have its

(i) greatest velocity and **(ii)** greatest displacement?



Example 5

A light plane must reach a speed of 35 m s^{-1} for take-off. Calculate,

(a) the time needed for the plane to reach its take-off speed if it accelerates uniformly from rest at 2.8 m s^{-2},

(b) the length of the runway that is needed.

Example 6

An object starts from rest and experiences a uniform acceleration of 0.50 m s^{-2} while moving down an inclined plane 9.0 m long. When it reaches the bottom, the object moves up another plane with uniform acceleration, where, after moving 15.0 m, it comes to rest.

Calculate,

(a) the speed of the object at the bottom of the first plane,

(b) the time taken for it to move down the first plane,

(c) the acceleration along the second plane, and

(d) the object's speed 8.0 m along the second plane.

Example 7

A stone is thrown vertically upward with a speed of 12 m s^{-1} from the edge of a cliff 100 m high.

(a) Calculate

(i) the time it takes to reach the bottom of the cliff,

(ii) its speed just before hitting the ground,

(iii) the total distance it travels.

(b) Sketch a graph showing how the velocity of the stone varies with time and label the axes with the appropriate values. Take $t = 0$ to be the time at which the stone is thrown.

3.3 Mass and linear momentum

❖ Mass and inertia

The inertia of a body can be described as its **reluctance to start moving**, or **to change its motion** once it has started.

The **mass** m of a body is the intrinsic property of a body which resists change in motion, i.e. it is a measure of the inertia of a body. The same force, when applied to a body of larger mass will cause a smaller change in motion.

It is important not to confuse weight with mass. The **weight** W of a body is the force experienced by a mass in a gravitational field.
It is an extrinsic property as it depends not just on the mass of the body, but also the gravitational field strength at the point where the body is.

❖ Momentum

In ancient times, the term 'motion' of a body was quantified based on the velocity of the body. However, during the late 17th and early 18th centuries, as the science of dynamics was being established, the kinematic concept of motion was deemed unsatisfactory.

Newton proposed that the quantity of motion should arise simultaneously from the velocity and 'quantity of matter' (i.e. mass) of the body. This is what is known as the **momentum** of a body.

Do you know?
The concept of momentum was actually first introduced by the French philosopher Jean Buridan (1295 – 1358), about four centuries before Issac Newton was born. Buridan described the quantity of motion as 'impetus', and that is a function of the quantity of matter and velocity of a body.

Definition

The **momentum** of a body is defined as the **product of its mass, *m*, and its velocity, *v*.**

$$p = mv$$

Since velocity is a vector, momentum is also a vector. Its direction follows the direction of the velocity. The S.I. unit of momentum is **kg m s^{-1} or N s**.

If we have a system of n bodies with masses m_1, m_2, e.t.c., the total momentum p of the system is the vector sum of the momenta of the individual bodies.

$$\begin{aligned} p &= p_1 + p_2 + \ldots + p_n \\ &= mv_1 + mv_2 + \ldots + mv_n \end{aligned}$$

Example 8

A body of A of mass 2.0 kg is moving to the left with a speed of 5.0 m s^{-1}. A body B of mass 4.0 kg is moving to the right with a speed of 2.5 m s^{-1}.

Calculate

(a) the momentum of A,

(b) the momentum of B,

the total momentum of the system of A and B.

3.4 Newton's Laws of motion

❖ **Newton's 1st Law**

The law states that a body continues in its state of rest or uniform motion in a straight line, unless a resultant external force acts on it.

The first law implies that

1. the **state of rest** requires no resultant force to maintain.
2. the **state of uniform velocity** also requires no resultant force to maintain.

The first law gives rise to the concept of *force*, whereby *force* is a quantity that can cause a body to undergo a change in velocity (or momentum).

> **Newton's 1st Law**
>
> A body continues in its state of rest or uniform motion in a straight line, unless a resultant external force acts on it.

❖ Newton's 2nd Law

The law states that the rate of change of momentum p of a body is proportional to the resultant force F_R acting on it and occurs in the direction of the force.

Mathematically,

$$F_R = k\frac{\mathrm{d}p}{\mathrm{d}t}$$

where k is the constant of proportionality.

In S.I. units, we set $k = 1$. Thus,

$$F_R = \frac{dp}{dt} = \frac{d(mv)}{dt}$$

The above mathematical description of Newton's 2nd law provides us with the concept of force – a force changes the motion of the body.

Newton's 2nd law implies that a body with a larger momentum would require either

- a larger force to stop (in the same time as one with a smaller momentum), or
- a longer time to stop with the same force.

Newton's 2nd Law
The rate of change of momentum of a body is proportional to the resultant force acting on it and occurs in the direction of the force.

Case 1: **Constant Mass**

When the resultant force F_R acts on a body of constant mass m,

$$F_R = m\frac{dv}{dt} = ma$$

The acceleration a of the body is in the same direction as the resultant force F_R.

One newton (N) is the force acting on a body of mass 1 kg which causes the body to accelerate at 1 m s^{-2} in the same direction of the force.

Case 2: **Constant Mass Flow**

When a continuous flow of mass experienced a change in velocity Δv, the resultant force F_R acting on the mass is given by

$$F_R = \Delta v\frac{\mathrm{d}m}{\mathrm{d}t}$$

❖ Newton's 3rd Law

The law states that whenever body A exerts a force $\vec{F}_{\text{A on B}}$ on body B, body B exerts an equal and opposite force $\vec{F}_{\text{B on A}}$ on body A.

Mathematically,

$$\left|\vec{F}_{\text{A on B}}\right| = \left|\vec{F}_{\text{B on A}}\right| \qquad \text{(equal in magnitude)}$$

$$\vec{F}_{\text{A on B}} = -\vec{F}_{\text{B on A}} \qquad \text{(opposite in direction)}$$

$\vec{F}_{\text{A on B}}$ and $\vec{F}_{\text{B on A}}$ are known as **action-reaction** pair of forces.

Newton's 3rd law has applications in every branch of physics and in everyday life. Walking or running would not be possible if Newton's 3rd law does not apply. The force exerted by the runner on the ground (Fig. 3.5) is equal in magnitude and opposite in direction to the force exerted by the ground on the runner which propels the runner forward.

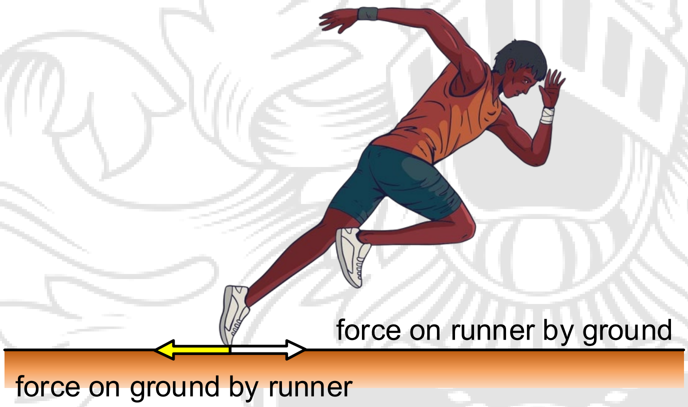


Fig. 3.5

On the other hand, although the normal contact force *N* exerted by the ground on the lady (Fig. 3.6) is equal in magnitude and opposite in direction to weight *W* of the lady, these two forces are not action-reaction pair of forces because the two forces are acting on the same body and they are not of the same type of forces.

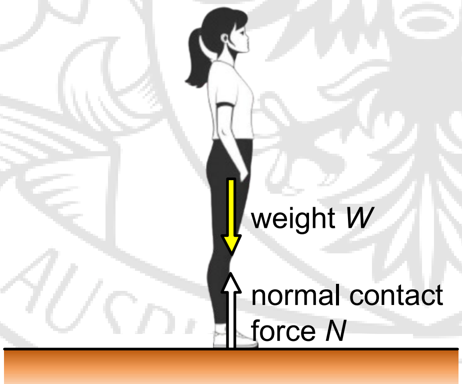


Fig. 3.6

For two forces to be an action-reaction pair of forces,

1. The two forces must be of the **same type**.
2. The two forces act on **different** bodies and they therefore **do not cancel** each other out.

Newton's 3rd Law

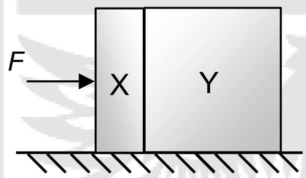
If a body A exerts a force on body B, then body B exerts an equal and opposite force on body A.

❖ Solving Dynamical Problems

1. Select a body or a system of bodies for analysis.
2. Draw a free-body diagram of the body or system showing all the forces acting on the body or system.
3. Select a direction to be positive. (It will be more convenient to take the direction of the acceleration as positive.)
4. Determine $F_{net} = \sum$(all forces)
5. Apply Newton's 2nd Law $F_{net} = ma$ to the chosen body or system.

Example 9

Two blocks X and Y, of masses M and $3M$ respectively, are accelerated along a smooth horizontal surface by a force F applied to block X as shown. In terms of F, determine the magnitude of the force exerted by block Y on block X during this acceleration.



Example 10

A helicopter of mass 1000 kg hovers by imparting a downward velocity v to the still air displaced by its rotating blades. The area swept out by the blades is 80 m^2.

Calculate the value of v.
(Density of air = 1.3 kg m^{-3}, g = 9.81 m s^{-2}.)